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# Effective action of matter fields in four-dimensional string orientifolds

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Abstract: We study various aspects of the Kähler metric for matter fields in $\mathcal{N} = 1, 2$ orientifold compactifications of type IIB string theory. The result has an infrared-divergent part which reproduces the field-theoretical anomalous dimensions, and a moduli-dependent part which comes from $\mathcal{N} = 2$ sectors of the orientifold. For the $\mathcal{N} = 2$ orientifolds, we also compute the disk amplitude for two matter fields on the boundary and a twisted closed string modulus in the bulk. Our results are in agreement with supersymmetry: the singlet under the $\mathrm{SU}(2)_R$ $R$-symmetry has vanishing coupling, while the coupling of the $\mathrm{SU}(2)_R$ triplet does not vanish.

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## 1. Introduction

In the past few years, due to the improved understanding of the role of D-branes in string theory, four-dimensional $\mathcal{N}=1$ Type IIB orientifold compactifications [1, 2] have received renewed attention. Compared to their weakly-coupled heterotic counterparts, which have been more thoroughly explored [3, 4], these models offer added flexibility, since the tree-level relations between gauge and string couplings or compactification and string scales are non universal. In particular, these models play an important part in brane-world scenarios (see [5] and references therein).

The purpose of this paper is to continue the work of [6, 7] on the determination of some parts of the effective action for these orientifold compactifications. While [7] discussed the gauge couplings, this article will focus on the study of the couplings of the matter fields. Let us first recall some general facts about the effective action of a four-dimensional field theory with $\mathcal{N}=1$ or $\mathcal{N}=2$ supersymmetry, emphasizing the main characteristics of interest here. More details can be found in [8, section 3], for instance.

The bosonic part of the effective lagrangian with at most two derivatives is given by the following expression:

$$\mathcal{L}_{\text{eff}} = -\frac{R}{2\kappa^2} + \frac{1}{2g_a^2(z)}\mathrm{tr}_a\big(F_{\mu\nu}F^{\mu\nu}\big) + \\ + \frac{\Theta_a(z)}{16\pi^2}\mathrm{tr}_a\big(F_{\mu\nu}\tilde{F}^{\mu\nu}\big) + \frac{1}{2}G_{ij}(z)D_\mu z^i D^\mu z^j + V(z)\,,$$

where $z$ are the scalar fields which parametrize a Kähler manifold of metric $G_{ij}$, and $V(z)$ is their potential. For $\mathcal{N}=1$ supergravity, this effective action is completely defined by the following functions:

- the Kähler potential $K(z,\bar{z})$ which determines the scalar metric $G_{i\bar{\jmath}} = \partial_i\partial_{\bar{\jmath}}K$. For $\mathcal{N}=1$ type I orientifolds without D5-branes, the matter field dependent part of the tree-level Kähler potential reads [2] $K = -\sum_{i=1}^{3}\log(\mathrm{Im}\,T^i + |\phi^i|^2/2)$ where $T^i$ is the Kähler structure and $\phi^i$ is the scalar matter field associated to the $i$-th torus,

- the analytic superpotential $W(z)$ which determines the part of the scalar potential associated to the F-terms: $V_{\mathrm{F}} = G^{i\bar{\jmath}}\partial_i W\partial_{\bar{\jmath}}W$, and which is not renormalized in perturbation theory,

- the analytic function $f_a(z)$ which gives the gauge couplings and the theta angles as $f_a(z) = \Theta_a(z)/8\pi^2 + i/g_a^2(z)$,

- the Fayet-Iliopoulos (FI) D-terms, due to the presence of anomalous U(1) factors of the gauge group. In type I compactifications, the anomaly-cancellation mechanism involves twisted Ramond-Ramond (R-R) fields and gives rise to D-term contributions to the scalar potential at tree level.

The calculation of the analytic function $f_a$ was the subject of ref. [7], where the tree-level couplings to twisted moduli and one-loop renormalization were extracted from annulus and Möbius strip diagrams, evaluated in a background magnetic field. Here, we will extend these results to other parts of the effective action, and in particular, we will find the one-loop renormalization of the Kähler metric of the matter fields charged under the gauge groups. However, we will perform direct calculations of the relevant scattering amplitudes, rather than using the background field method, for a reason we will explain below. The new results are as follows:

- in $\mathcal{N}=1$ sectors, the string oscillator modes do not decouple, and cut off the one-loop amplitude at the string scale $M_{\mathrm{S}}$. The infrared (IR) behavior of the one-loop string amplitude is governed by the field theoretical anomalous dimensions (example of the $\mathbb{T}^6/\mathbb{Z}_3$ model),

- in $\mathcal{N}=2$ sectors, only BPS states contribute to the D9-D9 annulus and to the Möbius strip amplitudes, and give rise to moduli-dependent threshold corrections for the matter field associated with the untwisted direction (example of the $\mathbb{T}^6/\mathbb{Z}_6'$ model),

- one-loop induced FI terms are absent for models with D5-branes and $\mathcal{N}=2$ sectors, generalizing the result of [9],

- tree-level D-terms, given by the coupling of twisted closed string states to bilinears in matter fields, contribute to the scalar potential. We have calculated them only in the context of $\mathcal{N}=2$ compactifications, where the twisted moduli space is simpler.

Compared to $\mathcal{N}=1$ compactifications, $\mathcal{N}=2$ supersymmetry imposes further restrictions on the effective action; in particular, at two-derivative order, there is no mixing between the hypermultiplets and the vector multiplets [10].[1] Moreover, for $\mathcal{N}=2$ type I compactifications, the four-dimensional dilaton belongs partly to a vector multiplet and partly to a hypermultiplet [11]. This should be contrasted with what happens in heterotic string theory, where the dilaton is in a vector multiplet, or with type II compactifications, where it is in a hypermultiplet.

The paper is organized as follows; in section 2, we describe the methods used to derive the tree-level couplings and one-loop corrections of the matter field metric for a general orbifold compactification. In particular, we give general expressions for the tree-level amplitude involving one closed, twisted NS-NS field and two open-string matter fields and needed to extract the FI D-terms, as outlined in the appendix of [12]. We also present the one-loop, two-point functions needed to obtained the one-loop corrections. Then, in section 3, we apply these methods to $K3\times\mathbb{T}^2$ orientifolds, showing which twisted moduli are effectively involved in the FI couplings. In section 4, we discuss the one-loop renormalization of the Kähler metric of matter fields in $\mathcal{N}=1$ compactifications; we verify that, for the $\mathcal{N}=1$ $\mathbb{Z}_3$ model, string theory reproduces the field theoretical anomalous dimensions. Finally, we study the $\mathcal{N}=1$ $\mathbb{Z}_6'$ model and we comment on the effective field theory interpretation.

## 2. General methods

We will consider four-dimensional $\mathbb{Z}_N$ orientifolds obtained by orbifolding the six-torus $\mathbb{T}^6$ by the twist operator $\theta=e^{2\pi i v_i J_i}$, with $J_i$ the generator of the rotation in the $i$-th complex plane. For a $\mathbb{Z}_N$ orientifold, $\theta^N=1$. Here $v_i=(v_1,v_2,v_3)$ is known as the twist vector. The twist $\theta^k$ also acts on the $n\times n$ Chan-Paton factors: this action is realized by $n\times n$ matrices, $\gamma^k$. We call the three complex coordinates of the

[1] Except those dictated by gauge symmetry [8, section 3].

six-torus $Z^i \equiv (X^{2i+2} + iX^{2i+3})/2$ for $i = 1, 2, 3$, and denote by $\phi^i$ the open string massless states associated to these directions, which correspond to Wilson lines for the D9-branes, or describe transverse positions of the D5-branes. The action of the orbifold on their Chan-Paton wave functions $\lambda_i$ is $\gamma^k \lambda_i (\gamma^k)^{-1} = e^{-2\pi i v_i} \lambda_i$ in order to obtain an invariant state. We will use Latin indices $i, j, \ldots$ for compact complex coordinates and Greek letters $\mu, \nu, \ldots$ for four-dimensional spacetime coordinates.

Denoting by $G_i$ the metric of the $i$-th torus $\mathbb{T}^i$ and by $V_i = \sqrt{G_i} M_S^2$ its volume, the gauge coupling constant on D9-branes is given by the inverse of the imaginary part of $S = a^{\text{R-R}} + iV_1V_2V_3\, e^{-\Phi_{10}}$. We also recall that the four-dimensional and ten-dimensional dilatons are related by $e^{-2\Phi_4} = e^{-2\Phi_{10}} V_1V_2V_3 M_S^{-6}$.

### 2.1 Background field method versus “dynamical branes”

In [7], the tree-level couplings to the closed twisted fields and the one-loop renormalization of the gauge couplings were extracted from a one-loop vacuum energy calculation in a background magnetic field, for various orientifold compactifications of type IIB string theory. The effect of this background field is to modify the boundary conditions for the open string [13]. Unfortunately, this method is not so useful for *twisted* complex coordinates, as we will explain below. First, we show that for coordinates left untwisted in some specific sectors, one can use a variant of the background field method, as follows. If $X^4$ and $X^5$ are the coordinates left untwisted, one takes the T-dual along one of these directions, say for instance $X^4$. This duality transforms D9-branes into D8-branes. Then, one gives an angle $\theta$ to one of these D8-branes in the $X^1X^4$ plane. The boundary conditions for a string stretched between this tilted D-brane and an untilted one become

$$\begin{cases} \partial_\sigma X^1(0,\tau) = 0\,, \\ X^4(0,\tau) = 0\,, \end{cases} \qquad \begin{cases} \partial_\sigma X^1(\pi,\tau) + \partial_\sigma X^4(\pi,\tau)\tan\theta = 0\,, \\ X^1(\pi,\tau)\tan\theta - X^4(\pi,\tau) = 0\,. \end{cases}$$

If we calculate the partition function in this background, the result turns out to be the same as the one obtained in [7] for the gauge fields.

Now consider instead T-dualizing in four compact, twisted directions to turn the D9-branes into D5-branes. For a twisted coordinate, giving an expectation value (linear in an untwisted coordinate) to the associated field means pulling the brane away from a fixed point, or in field theory language, moving on the Higgs branch of moduli space. We can pull branes away from the fixed point of a $\mathbb{Z}_N$ orbifold only in certain combinations of $N$ branes into a “dynamical brane”, which has no total charge under the twisted sector of the orbifold. To be precise, a dynamical brane away from a fixed point is made out of $N$ copies of the brane under the action of the orbifold group, $\mathbb{Z}_N$. (For the orientifold, we also have to include their images under world-sheet parity $\Omega$ [14, 15].) Labeling these branes by Chan-Paton indices $i = 1, \ldots, N$, their positions are given by $X(i)$ and the orbifold group acts as

$\theta(X(i)) = X(\gamma(i))$. Therefore, the Chan-Paton representation of $\theta$ on these branes is the permutation matrix:

$$\gamma = \begin{pmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & 0 & 0 & \cdots & 0 \end{pmatrix} .$$

We see that the dynamical brane is in the regular representation $R$ of the orbifold group. Now, it is easy to show that the boundary state associated to the representation $R$ is uncharged under the closed string twisted sector. Such boundary states have been constructed in [16]. Since $\mathrm{tr}_R(\gamma^k) = 0$ for $k = 1, \ldots, N-1$, the contributions of the twisted sectors to the boundary state describing a brane in the regular representation vanish. Therefore, branes away from fixed points have no couplings to the closed string twisted sector. In particular, this argument can be applied to branes in the magnetic field of [7]: when pulled away from a fixed point, these branes no longer have tree-level couplings between the twisted moduli and the gauge kinetic term. This also resolves an apparent paradox about the contribution of the classical action to these couplings. One could try to argue that for branes at a non-zero distance $|\phi|$ from an orbifold fixed point, the coupling would be suppressed by the classical action as $\exp(-|\phi|^2/\alpha')$. However, we know that such a term is not compatible with four-dimensional $\mathcal{N} = 2$ supersymmetry [10] since it involves a two-derivative coupling between hypermultiplets and vector multiplets. Happily, we have seen that this coupling is in fact absent for branes away from the fixed point, so there is no paradox. By the same token, we see that the background field method applied to the twisted coordinates can only give contributions from the untwisted sector of the orbifold. To obtain the twisted sector contributions to the couplings and renormalizations of these matter fields, we need to directly compute the relevant scattering amplitudes.

### 2.2 One-loop two-point function

To extract the one-loop renormalization of the wave function of the charged matter field, we calculate the even spin-structure part of the annulus and Möbius strip amplitudes with two open string vertices polarized in a twisted complex direction and inserted on the boundary, which is stuck on a D9-brane. The annulus with both ends on D9-branes reads:

$$\mathcal{A}_{99}(\phi^i, \phi^{\bar{\imath}}) = -\frac{1}{4N} \int_0^1 \frac{dq}{q} \int_0^q \frac{dz}{z} \int \frac{d^4p}{(2\pi)^4} \sum_{k=0}^{N-1} \mathrm{Tr}\Big[\theta^k V(\xi_1, p_1; z) V(\xi_2, p_2; q)\, q^{L_0}\Big],$$

where $L_0 = (p_\mu p^\mu + m^2)/2$ since we take $\alpha' = 1/2$. We will use the RNS formalism. The relevant vertex operators in the zero ghost picture are

$$V(\xi, p; z) = \lambda \xi_i \big(\partial X^i + i(p \cdot \psi)\psi^i\big) e^{ip\cdot X(z)}\,, \tag{2.1}$$

where $\xi_i$ is the polarization vector of the scalar $\phi^i$, and $\lambda$ is the Chan-Paton factor as above. Since we are interested in the one-loop renormalization of the kinetic terms of the matter fields, we will consider only the $\mathcal{O}(p^2)$ contribution to the two-point function. However, due to mass-shell conditions and momentum conservation, this amplitude vanishes. To extract information from this two-point function, we should relax one of these two conditions. To see which one, we recall that vertex operators of physical states must be BRST-invariant. For (2.1), the conditions are $p \cdot \xi_i = p \cdot p = 0$. On the other hand, momentum conservation comes from the integration of the zero modes which gives a function $\delta(\sum_i p_i)$, where the $p_i$ are the momenta of the external legs. Since the integration of zero modes is independent of the BRST conditions, we can relax momentum conservation and still have physical string amplitudes: $\delta \equiv p_1 \cdot p_2 \neq 0$. Although not completely justified,[2] we will see in the following that this procedure gives results which agree with the effective field theory description and with the heterotic counterpart [18], when available. Moreover, for the leading, $\delta$ independent term, one can justify the calculation by factorizing a four-point function, as in [19]. Finally, our method is in essence very similar to that used in ref. [3].

Doing the contractions, the annulus diagram reduces to:

$$\begin{aligned}\mathcal{A}_{99}\big(\phi^i, \phi^{\bar{\imath}}\big) = &-\frac{1}{2N}\, p_{1\mu} p_{2\nu}\, \xi^i \xi^{\bar{\imath}} \sum_{k=0}^{N-1} \mathrm{tr}\big(\gamma_9^k \lambda_1^\dagger \lambda_2\big) \mathrm{tr}\big(\gamma_9^k\big) \times \\ &\times \int_0^{i\infty} d\tau \sum_{\substack{\alpha,\beta=0,1/2 \\ \text{even}}} \frac{1}{2}\, \eta_{\alpha,\beta} Z_4^{\alpha,\beta}(\tau) Z_{\mathrm{int},k}^{\alpha,\beta}(\tau) \times \\ &\times \int_0^{\tau} d\nu\, e^{-\delta \langle X(z) X(q)\rangle} \big\langle \psi^\mu(z) \psi^\nu(q)\big\rangle^{\alpha,\beta} \big\langle \psi^i(z) \psi^{\bar{\imath}}(q)\big\rangle^{\alpha,\beta}, \end{aligned} \tag{2.2}$$

where we have introduced $q = e^{2\pi i\tau}$, $z = e^{2\pi i\nu}$ and $\eta_{\alpha,\beta} = (-1)^{2\alpha+2\beta+4\alpha\beta}$. The contribution of the zero modes and oscillators of the spacetime coordinates and ghosts to the partition function, denoted $Z_4^{\alpha,\beta}(\tau)$, and of the compact coordinates, $Z_{\mathrm{int}}^{\alpha,\beta}(\tau)$, are given in appendix A for all the models we will consider in this paper. The correlation functions are also given in this appendix. As usual, the Möbius strip amplitude is obtained by shifting the modular parameter $\tau$ by $1/2$ and taking into account the modification of the Chan-Paton traces. For models with D5-branes, the annulus amplitude with one boundary on the D9-branes and the other on the D5-branes may also contribute to the renormalization of the Kähler metric.

[2]This method has also been used in [11] to compute the renormalization of the Planck scale in orientifold compactifications. As explained there, a correct procedure is to start with a three-point amplitude with a U-modulus or the dilaton and the two other fields (for them gravitons, for us scalar fields) which are on-shell but have complex momenta. See also [17] for another alternative and more justified way to calculate these corrections.

We can formally expand the contributions of these amplitudes in powers of the momentum as

$$\mathcal{A}(\phi^i, \phi^{\bar{\imath}}) = \left(\zeta_{\phi^i} + \tilde{\gamma}_{\phi^i}\delta + \mathcal{O}(\delta^2)\right) \xi^i \xi^{\bar{\imath}} .$$

The first coefficient, $\zeta_{\phi^i}$, is the one-loop FI term while $\tilde{\gamma}_{\phi^i}$ will give the one-loop renormalization of the matter field metric. If there is an untwisted two-torus in a sector of the orbifold, these coefficients can depend explicitly through a logarithm on its moduli, as we will see in the following sections.

In the string frame, reinstating the tree-level contribution and the Einstein term, the two-derivative effective action for the matter fields reads:

$$\mathcal{L}^{(\mathrm{S})} = -\frac{1}{2\kappa^2} V_1 V_2 V_3 M_{\mathrm{S}}^{-6} e^{-2\Phi_{10}} R + \left(V_1 V_2 V_3 M_{\mathrm{S}}^{-6} e^{-\Phi_{10}} G_{i\bar{\imath}} + \tilde{\gamma}_{\phi^i}\delta_{i\bar{\imath}}\right) \partial_\mu \phi^i \partial^\mu \phi^{\bar{\imath}} ,$$

where $G_{i\bar{\imath}}$ is the tree-level metric, which, for correctly normalized string vertex operators, begins with $\delta_{i\bar{\imath}}$ (see [20, appendix B] for instance). To compare this string theory result with the field theory predictions, one has first to go to the Einstein frame. The correct redefinition of the metric in four dimensions is:

$$G^{(\mathrm{S})}_{\mu\nu} = e^{2\Phi_4} G^{(\mathrm{E})}_{\mu\nu} .$$

After this redefinition, the lagrangian density reads:

$$\mathcal{L}^{(\mathrm{E})} = -\frac{1}{2\kappa^2} R + \left( G_{i\bar{\imath}} + \frac{\tilde{\gamma}_{\phi^i}}{\operatorname{Im} S}\, \delta_{i\bar{\imath}} \right) e^{\Phi_{10}} \partial_\mu \phi^i \partial^\mu \phi^{\bar{\imath}} . \tag{2.3}$$

Here we ignore the one-loop universal correction to the Einstein term [11].

### 2.3 Disk amplitude and tree-level couplings

The tree-level couplings to closed twisted NS-NS fields can be obtained from a disk amplitude with two open vertices and one closed vertex.

The amplitude involves two charged matter vertices inserted on the boundary of the disk and one closed twisted vertex $V(\rho, k; z)$ in the interior:

$$\mathcal{A} = \int \frac{d^2 z\, dx_1\, dx_2}{V_{\mathrm{CKG}}} \operatorname{Tr} \left\langle V(\xi_1, p_1; x_1) V(\xi_2, p_2; x_2) V(\rho, k; z) \right\rangle ,$$

where $V_{\mathrm{CKG}}$ is the volume of the $\mathrm{PSL}(2,\mathbb{R})$ conformal Killing group of the disk. This $\mathrm{PSL}(2,\mathbb{R})$ invariance can be used to fix the positions of $V(\rho, k; z)$ and of one of the boundary operators; by a conformal transformation, we map the disk to the upper-half plane and choose $z = i$, $x_1 = x$ and $x_2 = -x$. Since the total superconformal ghost number of the disk is $-2$, we may choose the $(-1,-1)$ picture for the closed string vertex:

$$V(\rho, k; z) = \rho_{mn} e^{-\phi(z,\bar{z})} \psi^m(z) \sigma_k(z) \tilde{\psi}^n(\bar{z}) \sigma_k^\dagger(\bar{z}) e^{ik\cdot X(z,\bar{z})} , \tag{2.4}$$

where the $\sigma_k$, $\sigma_k^\dagger$ are the $\mathbb{Z}_N$ twist fields [21]. The relevant open string vertices are given by eq. (2.1). The correlators we need to evaluate the disk amplitude are given in appendix A. Then, it is easy to see that the amplitudes can expressed in term of the following integral:

$$I(\delta,\alpha) = 2^\delta \int_0^\infty dx\, x^{\delta-1}(x-i)^{-\delta+\alpha}(x+i)^{-\delta-\alpha}$$

for general $\delta$ and $\alpha$, which can be evaluated explicitly using hypergeometric functions; the result is

$$I(\delta,\alpha) = 2^\delta e^{i\pi\delta/2} B(\delta,\delta)_2F_1(\delta+\alpha,\delta;2\delta;2) = \sqrt{\pi} e^{-i\pi\alpha/2} \frac{\Gamma(\frac{\delta}{2})\Gamma(\frac{\delta+1}{2})}{\Gamma(\frac{\delta+1+\alpha}{2})\Gamma(\frac{\delta+1-\alpha}{2})}\,. \quad (2.5)$$

We will use this result in the following section.

## 3. $\mathcal{N}=2$ supersymmetry: $K3\times\mathbb{T}^2$ orientifolds

### 3.1 (No) one-loop renormalization of the hyperkähler metric

We start with the simplest models obtained by compactifying the six-dimensional $\mathbb{Z}_2$ orientifold [22, 14] (and its $\mathbb{Z}_N$ generalizations [15, 23]) to four dimensions. For these models, the twist vector is $\mathbf{v}=(1/N,-1/N,0)$. Tadpole cancellation requires that we introduce 32 D9-branes and, for $N$ even, 32 D5-branes. The six-dimensional $\mathcal{N}=(1,0)$ chiral hypermultiplet becomes a four-dimensional $\mathcal{N}=2$ hypermultiplet, while the vector multiplet gives an $\mathcal{N}=2$ vector multiplet whose complex scalar component comes from the directions along on the untwisted two-torus. Consequently, the renormalization of the metric of these latter scalars is related to the renormalization of the coupling constants. One can also see this result directly from a string calculation, using the background field method described in the previous section, which is valid for untwisted coordinates.

On the other hand, the four scalar fields which belong to the hypermultiplet correspond to twisted coordinates and require the direct methods outlined above. Let us begin with the one-loop amplitude. Using the correlators, the partition functions and the theta-function identity given in appendix A, one sees that the annulus amplitude vanishes:

$$\mathcal{A}_{99}(\phi^i,\phi^{\bar{\imath}}) = -\frac{\delta\xi^i\xi^{\bar{\imath}}}{8\pi^2 N}\sum_{k=0}^{N-1}\mathrm{tr}\big(\gamma_9^k\lambda_1^\dagger\lambda_2\big)\mathrm{tr}\big(\gamma_9^k\big)\left(2\sin\frac{\pi k}{N}\right)^2 \times$$
$$\times\int_0^{i\infty}\frac{d\tau}{2\tau^2}\Gamma^{(2)}(\tau)\frac{\vartheta_1(0|\tau)\eta(\tau)^{3\delta}}{\vartheta\big[{1/2 \atop 1/2+kv_i}\big](0|\tau)}\int_0^\tau d\nu e^{i\pi\delta\nu^2/\tau}\frac{\vartheta\big[{1/2 \atop 1/2+kv_i}\big](\nu|\tau)}{\vartheta_1(\nu|\tau)^{\delta+1}} = 0\,.$$

Similarly, one can easily see that the Möbius strip amplitude and, for $N$ even, the D9-D5 annulus also vanish. Moreover, the presence of a $\vartheta_1$ function with zero first argument shows that this result is related to the number of supersymmetries preserved by the compactification. To compare with the effective field theory prediction, we use eq. (2.3). For non-vanishing $\tilde{\gamma}$ coefficients, it predicts two-derivative couplings between the fields $\phi^i$ which are in hypermultiplets and $S$ which is in a vector multiplet. Since such terms are forbidden by $\mathcal{N}=2$ supersymmetry, we conclude that field theory also predicts the absence of one-loop renormalization of the hyperkähler metric.

### 3.2 Twisted tree-level couplings and Fayet-Iliopoulos terms

Now, we will derive the tree-level couplings of the closed string twisted fields to the charged hypermultiplets. To do this, we first need to recall the structure of the twisted moduli in $\mathbb{T}^4/\mathbb{Z}_N\times\mathbb{T}^2$ orientifold compactifications [15, 23, 12]. These twisted moduli can be interpreted as the Kaluza-Klein reduction of the ten-dimensional fields of type IIB string theory on supersymmetric two-cycles of $K3$, projected by the product of the orientation reversal operator $\Omega$ and an operator $J$ acting on the twisted sectors. As explained in [24], this operator $J$ exchanges sectors $k$ and $N-k$. For the untwisted sector, $J$ is just the identity. In type IIB, the dilaton, the metric and the R-R 2-form are $\Omega$-even while the NS-NS tensor, $B^{(2)}$, and the R-R scalar and 4-form are $\Omega$-odd. Therefore, the bosonic four-dimensional states are obtained by contracting the $\Omega$-even (odd) ten-dimensional fields with $J$-even (odd) combinations of harmonic forms from the $k$ and $N-k$ twisted sectors.

In the twisted NS-NS sector, the massless fields are given by the tensor product of left and right moving modes:

$$\begin{pmatrix}\psi^{\bar{1}}_{-1/2+k/N}\\ \psi^{2}_{-1/2+k/N}\end{pmatrix}\otimes\begin{pmatrix}\tilde{\psi}^{1}_{-1/2+k/N}\\ \tilde{\psi}^{\bar{2}}_{-1/2+k/N}\end{pmatrix}\big|p;k,\text{NS-NS}\big\rangle \quad \text{for } 1\le k<\frac{N}{2}\,,$$
$$\begin{pmatrix}\psi^{1}_{-1/2+(N-k)/N}\\ \psi^{\bar{2}}_{-1/2+(N-k)/N}\end{pmatrix}\otimes\begin{pmatrix}\tilde{\psi}^{\bar{1}}_{-1/2+(N-k)/N}\\ \tilde{\psi}^{2}_{-1/2+(N-k)/N}\end{pmatrix}\big|p;k,\text{NS-NS}\big\rangle \quad \text{for } \frac{N}{2}\le k\le N\,. \qquad (3.1)$$

As in [12], we decompose the rotational symmetry SO(4) which acts on the coordinates of the four-torus as $\mathrm{SU}(2)_L\times\mathrm{SU}(2)_R$ and define

$$\Psi=\begin{pmatrix}\psi^1 & -\psi^{\bar{2}}\\ \psi^2 & \psi^{\bar{1}}\end{pmatrix}.$$

One can classify the four twisted fields (3.1) according to their transformations under the $R$-symmetry group $\mathrm{SU}(2)_R$, and define a triplet $\mathrm{tr}(\Psi\vec{\sigma}\Psi^\dagger)$ and a singlet $\mathrm{tr}(\Psi\Psi^\dagger)$. The triplet state is associated to the complex structure and Kähler deformations of the manifold, whereas the singlet $b^{(0)}$ comes from the Kaluza-Klein reduction of the ten-dimensional $\Omega$-odd $B^{(2)}$ field on a vanishing supersymmetric two-cycle $\Sigma_k$ of the orbifold. The states are explicitly given in table 1.

| state | given by the action of |
|---|---|
| $b^{(0)}$ | $\psi^{\bar{1}}_{-1/2+k/N}\tilde{\psi}^{1}_{-1/2+k/N} + \psi^{2}_{-1/2+k/N}\tilde{\psi}^{\bar{2}}_{-1/2+k/N}$ |
| $\rho^3_k$ | $\psi^{\bar{1}}_{-1/2+k/N}\tilde{\psi}^{1}_{-1/2+k/N} - \psi^{2}_{-1/2+k/N}\tilde{\psi}^{\bar{2}}_{-1/2+k/N}$ |
| $\rho^+_k$ | $\psi^{2}_{-1/2+k/N}\tilde{\psi}^{1}_{-1/2+k/N}$ |
| $\rho^-_k$ | $\psi^{\bar{1}}_{-1/2+k/N}\tilde{\psi}^{\bar{2}}_{-1/2+k/N}$ |

**Table 1:** States obtained by action on the $k$-twisted NS-NS ground state (we have omitted the contributions of the $N-k$-sectors to these fields).

The R-R sector gives a six-dimensional anti-self-dual twisted 2-form and a twisted scalar, coming from the reduction of the R-R 4- and 2-form on $\Sigma_k$:

$$ {}^6C^{(2)}_k = \int^{10}_{\Sigma_k} C^{(4)}\,, \qquad {}^6C^{(0)}_k = \int^{10}_{\Sigma_k} C^{(2)}\,. $$

Reducing the anti-self-dual antisymmetric field on the two-torus gives a four-dimensional vector and an antisymmetric tensor (or, equivalently, its scalar dual).

The four fields (the NS-NS triplet and the R-R scalar ${}^6C^{(0)}_k$) which come from the Kaluza-Klein reduction of the $\Omega$-even sector fill out a hypermultiplet, while the singlet and its R-R partner ${}^6C^{(2)}_k$ give a $\mathcal{N}=2$ four-dimensional vector-tensor multiplet.

Since according to [10], there are no couplings between hypermultiplets and vector multiplets up to second order in derivatives, we expect that the amplitude with two charged hypermultiplets and the twisted singlet vanishes. On the other hand, the triplet can couple to the charged hypermultiplets and, indeed, it will correspond to an FI term as argued in [12]. We will now verify this claim by a direct calculation, using the method described in section 2.3.

Using (2.5), the disk amplitude with insertion of the singlet in the bulk and two charged open strings vertices (polarized in the twisted directions) on the boundary vanishes:

$$ \begin{aligned} \mathcal{A}\big(b^{(0)}_k,\phi_1,\phi_2\big) = b^{(0)}_k\xi^i\xi^{\bar{\imath}}\mathrm{tr}\big(\gamma_9^k\lambda_1^\dagger\lambda_2\big)\big(\delta I(\delta,2kv_i-1) - i(\delta-1+kv_i)I(\delta-1,2kv_i) +& \\ + ikv_iI(\delta-1,2kv_i-2)\big) = 0& \end{aligned} $$

as expected from the supersymmetry argument in the previous paragraph.

The amplitude with the triplet states and two matter fields is given by

$$ \begin{aligned} \mathcal{A}\big(\vec{\rho}_k,\phi_1,\phi_2\big) = \delta\Big(\rho^3_k\big(\xi^1\xi^{\bar{1}} + \xi^2\xi^{\bar{2}}\big) + \rho^+_k\xi^1\xi^2 + \rho^-_k\xi^{\bar{1}}\xi^{\bar{2}}\Big) \times& \\ \times\ \mathrm{tr}\big(\gamma_9^k\lambda_1^\dagger\lambda_2\big)I(\delta,2kv_i-1)& \end{aligned} $$

up to a numerical overall normalization which depends on $k$, and comes from the contractions of the twist fields fixed at the points $i$ and $-i$ on the double cover of the disk. Using the explicit expression of $I(\delta,2kv_i-1)$ and expanding the $\Gamma$ functions in $\delta$, we obtain a tree-level FI coupling between the twisted triplet and bilinears in the charged matter fields. Moreover, this amplitude also predicts the existence of a kinetic term coupling, and an infinite tower of derivative corrections as usual in string theory. However, these terms disappear when we take the on-shell limit ($\delta \to 0$) of the amplitude, so it is not safe to extrapolate to these orders. One the other hand, this procedure can be justified as in [19] for the momentum-independent term.

A final remark to conclude this section: the same method should allow us to recover the tree-level couplings between twisted field and gauge fields which were obtained in [7] by factorizing the one-loop amplitude in a background field; such direct tree-level calculation also clarifies the fact that, in the NS-NS sector, only the singlet propagates between branes in the magnetic field, a result which was not obvious within the factorization approach. However, as said before, one cannot really trust this computation since the amplitude vanishes on-shell. An alternative and more justifiable way to obtain this coupling is to use the background field method again. Let us just outline the procedure. The idea is to use a boundary state which corresponds to a brane in a constant magnetic field on the orbifold. Those can be constructed directly to reproduce the amplitudes given in [7] or, in the alternative T-dual picture, they can be obtained by a rotation in the spacetime directions of the twisted boundary states of [25, 16]. Then, the coupling of the twisted moduli to the magnetic field are calculated by evaluating the scalar product of these moduli with the boundary state. This argument shows that in fact, in the NS-NS sector, only the singlet couples to the magnetic field at quadratic order.

## 4. $\mathcal{N}=1$: anomalous dimensions and threshold corrections

### 4.1 Anomalous dimensions: the $\mathbb{Z}_3$ model

In this section, we will derive the kinetic terms at one-loop for the charged matter fields in a $\mathbb{Z}_N$ orbifold with $N$ a prime integer. Since there are no order two twist elements, these compactifications have only $\mathcal{N}=1$ sectors and no D5-branes. The one-loop two-point function is given by the sum of the annulus and Möbius strip amplitudes:

$$\mathcal{A}(\phi^i,\phi^{\bar\imath}) \equiv -\frac{1}{2N}\sum_{k=1}^{N-1}\int_0^{i\infty}\frac{d\tau}{\tau}\Big(\mathcal{A}_{99}^{(k)}(q)+\mathcal{M}_9^{(k)}(-q)\Big)\,.$$

We have omitted the $k=0$ sector, which corresponds to the contribution of the $\mathcal{N}=4$ supersymmetric open string spectrum and therefore does not contribute to wave function renormalization. Using the "amplitude toolbox" given in appendix A, the two-point function becomes

$$\begin{aligned}\mathcal{A}_{99}^{(k)}(q) = &-\frac{\delta\,\xi^i\xi^{\bar\imath}}{4\pi^2}\mathrm{tr}\big(\gamma_9^k\lambda_1^\dagger\lambda_2\big)\mathrm{tr}\big(\gamma_9^k\big)\prod_{j=1}^{3}(-2\sin\pi k v_j)\,\times\\ &\times\frac{1}{2\tau}\frac{\eta(\tau)^{3(1+\delta)}}{\vartheta\big[{}^{1/2}_{1/2+kv_i}\big](0|\tau)}\int_0^\tau d\nu e^{i\pi\delta\nu^2/\tau}\frac{\vartheta\big[{}^{1/2}_{1/2+kv_i}\big](\nu|\tau)}{\vartheta_1(\nu|\tau)^{\delta+1}}\end{aligned} \tag{4.1}$$

for the annulus contribution and

$$\mathcal{M}_9^{(k)}(-q) = \frac{\delta\xi^i\xi^{\bar{\imath}}}{4\pi^2}\mathrm{tr}\big(\gamma_9^{2k}\lambda_1^\dagger\lambda_2\big)\prod_{j=1}^{3}(-2\sin\pi k v_j)\times$$
$$\times\frac{1}{2\tau}\frac{\eta(\tau+1/2)^{3(1+\delta)}}{\vartheta\big[{}^{1/2}_{1/2+kv_i}\big](0|\tau+1/2)}\int_0^{2\tau}d\nu\; e^{i\pi\delta\nu^2/\tau}\frac{\vartheta\big[{}^{1/2}_{1/2+kv_i}\big](\nu|\tau+1/2)}{\vartheta_1(\nu|\tau+1/2)^{\delta+1}} \qquad (4.2)$$

for the Möbius strip amplitude. We observe that the string oscillators do not decouple and, therefore, contribute to the renormalization.

Now, we will compare this result with the field theory prediction of the anomalous dimensions for the $\mathbb{Z}_3$ model [1]. This orientifold is defined by the twist vector $\mathbf{v} = (1/3, 1/3, -2/3)$. The comparison requires extracting the infrared contribution of the string amplitudes (4.1) and (4.2) in the open channel. To do this, we write the theta functions as products and take the limit $q = e^{2i\pi\tau} \to 0$. Expanding the integrand in series of powers of $\delta$ (recall that we are only interested in order one in $\delta$), the integral over $\nu$ can be done explicitly. Define $\alpha_i = e^{2\pi i k v_i}$. The result for the annulus is

$$\lim_{q\to 0}\mathcal{A}_{99}^{(k)}(q) = -\frac{\delta\,\xi^i\xi^{\bar{\imath}}}{8\pi^2}\mathrm{tr}\big(\gamma_9^k\lambda_1^\dagger\lambda_2\big)\mathrm{tr}(\gamma_9^k)\prod_{j=1}^{3}(-2\sin\pi k v_j)\times$$
$$\times\left[\frac{\bar{\alpha}_i^k-1}{2\pi\tau\delta}+\frac{i\bar{\alpha}_i^k}{1-\bar{\alpha}_i^k}+\mathcal{O}(\delta)\right]$$

and for the Möbius strip,

$$\lim_{q\to 0}\mathcal{M}_9^{(k)}(-q) = \frac{\delta\xi^i\xi^{\bar{\imath}}}{8\pi^2}\mathrm{tr}(\gamma_9^{2k}\lambda_1^\dagger\lambda_2)\prod_{j=1}^{3}(-2\sin\pi k v_j)\times$$
$$\times\left[\frac{\bar{\alpha}_i^{2k}-1}{2\pi\tau\delta}+\frac{i(\bar{\alpha}_i^k+\bar{\alpha}_i^{2k})}{1-\bar{\alpha}_i^k}+\mathcal{O}(\delta)\right].$$

To add these contributions, one has to rescale the modular parameter of the Möbius strip relative to the cylinder. The correct rescaling is obtained by normalizing the proper time in the closed string channel ($\ell$) through the closed string propagators [26, 6]. The relation between $\ell$ and the proper times in the direct channel for the annulus and Möbius strip is $\tau_M = \tau_A/4 = 1/4\ell$. Moreover, for the $\mathbb{Z}_3$ orientifold, $\mathrm{tr}(\gamma_9^k) = -4$ and $\prod_{j=1}^3(-2\sin\pi k v_j) = -(-1)^k 3\sqrt{3}$.

With this in mind, one sees upon adding the two contributions that the leading ($\delta$-independent) term vanishes. This was already observed in [9] where the one-loop FI term was calculated in the GS formalism, and shown to vanish. Cutting off the integral by introducing the infrared regulator $t = -2i\tau_A \le 1/\mu^2$, we find the infrared behavior:

$$\mathcal{A} = -\frac{3i\sqrt{3}}{32\pi^2}\,\delta\xi^i\xi^{\bar{\imath}}\sum_{k=1}^{2}\frac{1}{1-\bar{\alpha}_i^k}\,\mathrm{tr}\big(\gamma_9^k\lambda_1^\dagger\lambda_2\big)\ln\frac{\mu^2}{M_I^2}\,. \qquad (4.3)$$

To evaluate the Chan-Paton trace, we need to introduce a little bit more details. As explained in [1], the massless matter content of the $\mathbb{Z}_3$ model is given by three copies of the $(1,\overline{66})_{-2}$ and $(8,12)_1$ representations of an $\mathrm{SO}(8)\times\mathrm{SU}(12)\times\mathrm{U}(1)$ gauge group (the superscripts denote the U(1) charge).

To be explicit, we introduce the generators $\sigma_{ar}$ and $\tau_{rs}$, for $a=1,\ldots,8$ and $r,s=1,\ldots,12$, normalized such that $\mathrm{tr}(\sigma^T_{ar}\sigma_{bs})=\frac{1}{2}\,\delta_{ab}\delta_{rs}$ and $\mathrm{tr}(\tau_{pq}\tau_{rs})=\frac{1}{2}\,(\delta_{ps}\delta_{qr}-\delta_{pr}\delta_{qs})$. In this basis, the matter fields can be written as $\phi^i\lambda_i=2\psi^{ir}_a\sigma_{ar}$ for the $(8,12)$ representation and $\phi^i\lambda_i=2\chi^i_{rs}\tau_{rs}$ for the $(1,\overline{66})$. Starting from the ten-dimensional SYM theory and performing the Kaluza-Klein reduction to four dimensions, this gives correctly normalized kinetic terms for the complex matter fields. The Chan-Paton trace in (4.3) can now be evaluated:

$$\Delta\mathcal{L}^{(\mathrm{S})}_{\text{one-loop}}=\frac{3}{32\pi^2}\ln\frac{\mu^2}{M_I^2}\left(\partial_\mu\psi^{ir}_a\partial^\mu\psi^{ir\dagger}_a-2\partial_\mu\chi^i_{rs}\partial^\mu\chi^{i\dagger}_{rs}\right)\tag{4.4}$$

for the one-loop renormalization of these fields in the string frame. To compare this result with the field theory prediction, we first go to the Einstein frame. The IR divergent terms can therefore be summarized by the lagrangian (2.3), with $\tilde{\gamma}_\psi=\frac{3}{32\pi^2}\ln(\mu^2/M_I^2)$ and $\tilde{\gamma}_\chi=-\frac{3}{16\pi^2}\ln(\mu^2/M_I^2)$. Now, we will show that these coefficients are related to the anomalous dimensions $\gamma$ of the matter fields according to $\gamma\ln(\mu^2/M_I^2)=\tilde{\gamma}/\,\mathrm{Im}\,S$.

In an $\mathcal{N}=1$ SYM theory with a simple gauge group and a generic superpotential,

$$\mathcal{W}=\frac{1}{6}\,\lambda^{abc}_{ijk}\,\phi^i_a\,\phi^j_b\,\phi^k_c\,,$$

where $i,j,k$ are family indices and $a,b,c$ group indices (for us, the family indices will label the three complex planes), the anomalous dimensions of the matter fields $\phi^i_a$ are given by the formula [27]

$$(\gamma_i{}^j)^a{}_b=-\frac{1}{16\pi^2}\left(2g^2C_2(R_a)\,\delta_i{}^j\,\delta^a{}_b-\sum_{kl,cd}\lambda^{acd}_{ikl}\lambda^{jkl}_{bcd}\right),$$

where $C_2(R_a)$ is the quadratic Casimir of the representation $R_a$ and $\lambda^{ijk}_{abc}=\lambda^{abc*}_{ijk}$. This formula can be easily generalized to semi-simple groups with U(1) factors. In particular, for the $\mathbb{Z}_3$ model, where the superpotential is given by

$$\mathcal{W}=\sqrt{\frac{1}{2\,\mathrm{Im}\,S}}\epsilon_{ijk}\psi^i\chi^j\psi^k$$

one finds

$$(\gamma_\psi)_i{}^j=-\frac{1}{16\pi^2}\left(\left(2g^2_{\mathrm{SU}(12)}C_2^{\mathrm{SU}(12)}(12)+2g^2_{\mathrm{SO}(8)}C_2^{\mathrm{SO}(8)}(8)+g^2_{\mathrm{U}(1)}\right)-\frac{11}{\mathrm{Im}\,S}\right)\delta^j_i\,,$$
$$(\gamma_\chi)_i{}^j=-\frac{1}{16\pi^2}\left(\left(2g^2_{\mathrm{SU}(12)}C_2^{\mathrm{SU}(12)}(66)+4g^2_{\mathrm{U}(1)}\right)-\frac{8}{\mathrm{Im}\,S}\right)\delta^j_i\,,$$

where we have suppressed a multitude of Kronecker deltas in the group indices. The coupling constants for the gauge groups are $g^2_{\mathrm{SO}(8)} = 1/\operatorname{Im} S$, $g^2_{\mathrm{SU}(12)} = 1/(2\operatorname{Im} S)$, $g^2_{\mathrm{U}(1)} = 1/(24\operatorname{Im} S)$ so the final result is

$$(\gamma_\psi)_i{}^j = \frac{3\delta_i{}^j}{32\pi^2\operatorname{Im} S}\,, \qquad (\gamma_\chi)_i{}^j = -\frac{3\delta_i{}^j}{16\pi^2\operatorname{Im} S}\,,$$

in agreement with the string theory result.

### 4.2 One-loop Fayet-Iliopoulos term: the $\mathbb{Z}_6'$ model

The $\mathbb{Z}_6'$ model [2] is defined by the twist vector $\mathbf{v} = (1/6, -1/2, 1/3)$. The requirement of tadpole cancellation forces us to introduce 32 D9-branes and also 32 D5-branes filling the space transverse to the first and second complex planes. We refer the reader to section 5 of [7] for more details on this model; here we only summarize the characteristics needed to compute the one-loop kinetic term of the matter fields. The model contains an $\mathcal{N} = 4$ sector ($\theta^0$), two $\mathcal{N} = 1$ sectors ($\theta^1, \theta^5$) and three $\mathcal{N} = 2$ sectors ($\theta^2, \theta^3, \theta^4$). For $k = 2,4$ (resp., $k = 3$), the second (resp., third) complex plane is untwisted. It is important to note that since the D5-branes fill the third complex plane but not the first and the second, 9–5 strings in the $k = 3$ sector enjoy the full $\mathcal{N} = 2$ supersymmetry, whereas 9–5 strings in the $k = 2,4$ sectors see $\mathcal{N} = 1$ supersymmetry only. In the former sector, the D5-branes are transverse only to twisted directions, and thus break no supersymmetry that was not already broken by the D9-branes and the action of the orientifold.

The amplitudes for the two-point function of the scalar $\phi^1$ are

$$\begin{aligned}
\mathcal{A}_{99}(\phi^1,\phi^{\bar 1}) &= \frac{\delta\xi^1\xi^{\bar 1}}{96\pi^2}\sum_{k=1,5}\operatorname{tr}\big(\gamma_9^k\lambda_1^\dagger\lambda_2\big)\ \operatorname{tr}(\gamma_9^k)\prod_{j=1}^3(-2\sin\pi k v_j)\ \times\\
&\quad\times\int_0^{i\infty}\frac{d\tau}{\tau^2}\,\frac{\eta(\tau)^{3(1+\delta)}}{\vartheta\big[{}^{1/2}_{1/2+k/6}\big](0|\tau)}\int_0^\tau d\nu\ e^{i\pi\delta\nu^2/\tau}\,\frac{\vartheta\big[{}^{1/2}_{1/2+k/6}\big](\nu|\tau)}{\vartheta_1(\nu|\tau)^{\delta+1}}\,,\\
\mathcal{M}_9(\phi^1,\phi^{\bar 1}) &= -\frac{\delta\xi^1\xi^{\bar 1}}{96\pi^2}\sum_{k=1,5}\operatorname{tr}\big(\gamma_9^{2k}\lambda_1^\dagger\lambda_2\big)\prod_{j=1}^3(-2\sin\pi k v_j)\ \times\\
&\quad\times\int_0^{i\infty}\frac{d\tau}{\tau^2}\,\frac{\eta(\tau+1/2)^{3(1+\delta)}}{\vartheta\big[{}^{1/2}_{1/2+k/6}\big](0|\tau+1/2)}\int_0^{2\tau}d\nu e^{i\pi\delta\nu^2/\tau}\,\frac{\vartheta\big[{}^{1/2}_{1/2+k/6}\big](\nu|\tau+1/2)}{\vartheta_1(\nu|\tau+1/2)^{\delta+1}}\,,\\
\mathcal{A}_{95}(\phi^1,\phi^{\bar 1}) &= -\frac{\delta\xi^1\xi^{\bar 1}}{48\pi^2}\sum_{k=1,2,4,5}\operatorname{tr}\big(\gamma_9^k\lambda_1^\dagger\lambda_2\big)\operatorname{tr}(\gamma_5^k)\sin\frac{\pi k}{3}\ \times\\
&\quad\times\int_0^{i\infty}\frac{d\tau}{\tau^2}\,\frac{\eta(\tau)^{3(1+\delta)}}{\vartheta\big[{}^{0}_{1/2+k/6}\big](0|\tau)}\int_0^\tau d\nu\ e^{i\pi\delta\nu^2/\tau}\,\frac{\vartheta\big[{}^{0}_{1/2+k/6}\big](\nu|\tau)}{\vartheta_1(\nu|\tau)^{\delta+1}}\,.
\end{aligned}$$

This field $\phi^1$, which comes from a complex plane twisted by all sectors of the orbifold (i.e. $kv_1 \notin \mathbb{Z}$ for all $k$), only receives contributions from the $\mathcal{N} = 1$ sector for the

$\mathcal{A}_{99}$ and $\mathcal{M}_9$ amplitudes. For these two diagrams, the contributions of the $\mathcal{N}=2$ sectors vanish, just like we already observed for the scalars of the $\mathcal{N}=2$ hypermultiplets in $K3\times\mathbb{T}^2$ compactifications. Notice that, due to the tadpole conditions $\mathrm{tr}(\gamma_9^k)=\mathrm{tr}(\gamma_5^k)=0$ for $k=1,3,5$, the amplitude $\mathcal{A}_{99}$ vanishes identically, and one immediately sees that there is no one-loop FI terms proportional to $\mathrm{tr}(\gamma_9^k\lambda_1^\dagger\lambda_2)$ with $k$ odd, since the Möbius strip can only contribute to even powers in $\gamma^k$. Further, one can perform an expansion as already performed for the $\mathbb{Z}_3$ model, using now the conditions $\mathrm{tr}(\gamma_5^2)=-\mathrm{tr}(\gamma_5^4)=-8$ and $\gamma_9^6=-1$ [2]. The result is that the contribution of the $\mathcal{N}=1$, $k=1,5$ sectors of the Möbius strip to the would-be one-loop FI term is cancelled by the $k=2,4$ sectors of the 9–5 annulus, which are actually $\mathcal{N}=1$ for this 9–5 amplitude only, as noted above.

The FI D-term would have looked like $\zeta_{\rm FI}^2\sim\Lambda_{\rm UV}\,\mathrm{tr}\,Q_{\rm U(1)}$ for a UV cutoff $\Lambda_{\rm UV}$, which would have generated a mass term for the charged scalar, proportional to its U(1) charge. We thus see that this term is not generated. One can easily check that such mass terms are also absent for the two other scalar fields, $\phi^2$ and $\phi^3$. Finally, as in the previous section, the one-loop renormalization of this field is given by its field theoretical $\mathcal{N}=1$ anomalous dimensions.

### 4.3 Threshold corrections: the $\mathbb{Z}_6'$ model

The scalar $\phi^2$ and its complex conjugate $\phi^{\bar 2}$ come from a plane which is untwisted in the $k=2,4$ sectors. The relevant one-loop two-point amplitudes are

$$\begin{aligned}
\mathcal{A}_{99}\big(\phi^2,\phi^{\bar 2}\big) = \frac{\delta\xi^2\xi^{\bar 2}}{96\pi^2}\Bigg[&\sum_{k=1,5}\mathrm{tr}\left(\gamma_9^k\lambda_1^\dagger\lambda_2\right)\mathrm{tr}\left(\gamma_9^k\right)\prod_{j=1}^{3}(-2\sin\pi k v_j)\times\\
&\times\int_0^{i\infty}\frac{d\tau}{\tau^2}\frac{\eta(\tau)^{3(1+\delta)}}{\vartheta\left[{1/2 \atop 1/2-k/2}\right](0|\tau)}\int_0^{\tau}d\nu e^{i\pi\delta\nu^2/\tau}\frac{\vartheta\left[{1/2 \atop 1/2-k/2}\right](\nu|\tau)}{\vartheta_1(\nu|\tau)^{\delta+1}}+\\
&+\sum_{k=2,4}\mathrm{tr}\big(\gamma_9^k\lambda_1^\dagger\lambda_2\big)\mathrm{tr}\big(\gamma_9^k\big)\prod_{j=1,3}(2\sin\pi k v_j)\int_0^{i\infty}\frac{d\tau}{\tau}\Gamma_2^{(2)}(\tau)\Bigg],
\end{aligned}$$

$$\begin{aligned}
\mathcal{M}_9\big(\phi^2,\phi^{\bar 2}\big) = -\frac{\delta\,\xi^2\xi^{\bar 2}}{96\pi^2}\Bigg[&\sum_{k=1,5}\mathrm{tr}\big(\gamma_9^{2k}\lambda_1^\dagger\lambda_2\big)\prod_{j=1}^{3}(-2\sin\pi k v_j)\times\\
&\times\int_0^{i\infty}\frac{d\tau}{\tau^2}\frac{\eta(\tau+1/2)^{3(1+\delta)}}{\vartheta\left[{1/2 \atop 1/2-k/2}\right](0|\tau+1/2)}\times\\
&\times\int_0^{2\tau}d\nu e^{i\pi\delta\nu^2/\tau}\frac{\vartheta\left[{1/2 \atop 1/2-k/2}\right](\nu|\tau+1/2)}{\vartheta_1(\nu|\tau+1/2)^{\delta+1}}+\\
&+2\sum_{k=2,4}\mathrm{tr}\big(\gamma_9^{2k}\lambda_1^\dagger\lambda_2\big)\prod_{j=1,3}(2\sin\pi k v_j)\int_0^{i\infty}\frac{d\tau}{\tau}\,\Gamma_2^{(2)}(\tau)\Bigg],
\end{aligned}$$

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$$\mathcal{A}_{95}\left(\phi^2, \phi^{\bar{2}}\right) = -\frac{\delta\, \xi^2 \xi^{\bar{2}}}{48\pi^2} \sum_{k=1,2,4,5} \mathrm{tr}\left(\gamma_9^k \lambda_1^\dagger \lambda_2\right) \mathrm{tr}(\gamma_5^k) \sin\frac{\pi k}{3} \times$$
$$\times \int_0^{i\infty} \frac{d\tau}{\tau^2} \frac{\eta(\tau)^{3(1+\delta)}}{\vartheta\left[{0 \atop 1/2-k/2}\right](0|\tau)} \int_0^\tau d\nu\; e^{i\pi\delta\nu^2/\tau} \frac{\vartheta\left[{0 \atop 1/2-k/2}\right](\nu|\tau)}{\vartheta_1(\nu|\tau)^{\delta+1}}\,.$$

We observe that, besides the field theoretical $\mathcal{N}=1$ renormalization running up to the string scale, the corrections given by the $\mathcal{N}=2$ sectors depend on the geometric moduli of the complex planes in the same way as for the gauge bosons. Indeed, in the sectors $k=2,4$ where the scalars are untwisted, one can use the background field method suggested at the beginning of section 2 to obtain a result identical to that of the gauge bosons. Notice that this argument also shows that at tree-level, the twisted NS-NS field in the $k=2,4$ sectors couples to the kinetic term of this matter field $\phi^2$ in the same way as the gauge field does. We can explain this phenomenon as follows: we start with a four-dimensional orientifold compactification with a twist vector defined as $\mathbf{v}' = 2\mathbf{v} = (1/3, -1, 2/3)$ which generates a $\mathbb{Z}_3$ subgroup of the original $\mathbb{Z}_6'$. The result is an $\mathcal{N}=2$, $\mathbb{Z}_3$ orbifold of the family we studied in the first part of this section. However, it will also have D5-branes which, as we have already seen, are crucial for the absence of one-loop FI terms, but do not fill completely the space transverse to $K3$. This $\mathcal{N}=2$ orbifold leaves the second complex plane untwisted. As argued above, the scalar fields associated to this plane belong to vector multiplets, and so they have the same one-loop renormalization and couplings to the twisted fields of the orbifold. The projection on $\mathbb{Z}_6'$ invariant states eliminates some of the fields but these couplings survive.

We now describe in more details the threshold corrections coming from $\mathcal{N}=2$ sectors. We denote by $U$ the complex structure of the second two-torus:

$$U = \frac{G_2^{12} + i\sqrt{G_2}}{G_2^{11}}\,.$$

The threshold corrections due to $\mathcal{N}=2$ sectors are given by the sum of $\mathcal{A}_{99}^{(k)}$ and $\mathcal{M}_9^{(k)}$ for $k=2,4$:

$$\frac{1}{6(2\pi)^2} \sum_{k=2,4} \prod_{j=1,3} (\sin \pi k v_j) \left[\mathrm{tr}\left(\gamma_9^k \lambda_1^\dagger \lambda_2\right) \mathrm{tr}(\gamma_9^k) - 2\mathrm{tr}\left(\gamma_9^{2k} \lambda_1^\dagger \lambda_2\right)\right] \int \frac{d\tau}{\tau} \Gamma_2^{(2)}(\tau) =$$
$$= \frac{3}{(4\pi)^2} \sum_{k=2,4} \mathrm{tr}\left(\gamma_9^k \lambda_1^\dagger \lambda_2\right) \left[\ln\left(\sqrt{G_2}\, \mathrm{Im}\, U \mu^2\right) + 4\, \mathrm{Re} \ln \eta(U)\right]. \qquad (4.5)$$

If the Chan-Paton matrices $\lambda_i$ had been in the adjoint representation of the group, the coefficients of these threshold corrections would have been the $\mathcal{N}=2$ effective theory beta functions of the corresponding sector. These corrections reproduce the heterotic ones only in the limit $\mathrm{Im}\, T \to 0$. Non-perturbative corrections are needed to reproduce the complete threshold dependence on $T$ [18] which, on the heterotic side, is just the Kähler modulus of the torus, but on the type I side depends on the ten-dimensional string coupling constant as $T = b_2^{\text{R-R}} + i\sqrt{G_2} M_S^2 e^{-\Phi_{10}}$.

We have obtained similar results for the scalar $\phi^3$, except for one important point, its tree-level couplings to the twisted moduli of the $\mathcal{N}=2$ sector. Indeed, as noticed in [7], the corresponding twisted field belongs to a hypermultiplet — which cannot couple to kinetic terms of non-abelian gauge fields because of $\mathcal{N}=2$ supersymmetry — and the complex field $\phi^3$ is in the vector multiplet of the $\mathcal{N}=2$, $\mathbb{Z}_2$ orientifold generated by $\mathbf{v}' = 3\mathbf{v} = (1/2, -3/2, 1)$. On the other hand, it can couple to $\mathcal{N}=1$ twisted fields; such couplings should be obtained using the method described in the final part of the previous section. Finally, using tadpole conditions, one shows that its threshold corrections come from the $\mathcal{M}_9^{(3)}$ and $\mathcal{A}_{95}^{(0)}$ amplitudes and depends on the complex structure of the third two-torus, a result which can be explained as for $\phi^2$.

## 5. Conclusions and discussion

In this article, we have investigated some parts of the effective action of four-dimensional type I compactifications, focusing in particular on the one-loop renormalization of the Kähler metric and the tree-level couplings between charged matter fields and twisted moduli.

For the renormalization of the Kähler metric, the general picture is the following: on the one hand, $\mathcal{N}=1$ sectors yield moduli-independent corrections to the metric, and hence to the physical Yukawa couplings. Due to the reduced number of supersymmetries, the string oscillators do not decouple, and the renormalization constant of the charged field is given by infrared logarithmic corrections, independent of the volume of the compact space, cut off at the string scale $M_S$ and with a coefficient given by the field theory $\gamma$ functions. On the other hand, moduli-dependent threshold corrections arise in $\mathcal{N}=2$ sectors, for scalar fields associated to the plane left invariant by the twist operator in these sectors. The phenomenological use of this kind of corrections in models with low string scale is discussed in [31]. For a rectangular untwisted torus of radii $R_1$, $R_2$, these corrections are proportional to $\ln(\mu^2 R_1 R_2) + f(R_1/R_2)$, where $\mu$ is the infrared scale and $f$ diverges linearly when $R_1 \gg R_2$. Otherwise, the scalars associated to twisted plane are not renormalized. The contribution of the D9-D5 annulus amplitude is special, in the sense that in the $\mathcal{N}=2$ sectors where the D5-brane is wrapped around twisted directions and therefore breaks half of these $\mathcal{N}=2$ supersymmetries, it gives corrections similar to those of $\mathcal{N}=1$ sectors.

Within this computation of one-loop amplitudes, we have also recovered the result of [9] on the absence of one-loop induced Fayet-Iliopoulos term and generalized it to $\mathcal{N}=1$ orientifolds with $\mathcal{N}=2$ sectors and D5-branes. This vanishing occurs because of the cancellation between contributions of worldsheets with different topology and of different sectors. In particular, the presence of D5-branes is crucial in this mechanism. This cancellation is related to the absence of twisted R-R tadpoles in these models.

Finally, we have calculated explicitly tree-level couplings between the twisted fields of the orbifold and charged fields for $K3 \times \mathbb{T}^2$ orientifolds. In agreement to supersymmetry predictions, we have obtained couplings between charged matter fields and the three NS-NS twisted moduli which, with a R-R twisted scalar, make up a hypermultiplet and which transform as a triplet of the $R$-symmetry group $\mathrm{SU}(2)_R$. These NS-NS fields are also the blow-up modes of the orbifold. On the other hand, the coupling constant contains a tree-level part proportional to the scalar component of $\mathcal{N}=2$ twisted vector-tensor multiplet, which is also a singlet under $\mathrm{SU}(2)_R$. The CP-odd counterpart of these couplings have been investigated in detail in [28], where they were extracted by factorization of one-loop amplitudes in the odd spin-structure. The result is that the $\mathrm{tr}(F \wedge F)$ couples to the twisted R-R tensor which is in a $D=6$, $\mathcal{N}=(1,0)$ tensor multiplet while the U(1) field couples to the R-R scalar field as $\int {}^6 C_k^{(0)} \mathrm{tr}\gamma^{(k)} F$. The supersymmetric partner of the Chern-Simons coupling of the tensor field is the tree-level coupling between the singlet $b_k^{(0)}$ and $F^2$ while the counterpart of the other term is given by the Fayet-Iliopoulos D-terms: $\int \vec{\rho} \cdot \vec{D}$. Integrating out the auxiliary fields $\vec{D}$ gives, as explained in [12], the coupling between bilinears in the charged matter field and the twisted triplet that we have calculated directly in string theory.

Such disk calculations should easily generalize to $\mathcal{N}=1$ compactifications, for which the twisted moduli space structure was described in [29], for instance. Again, the CP-odd partners of these couplings have been analyzed in [30]; for twisted sectors without fixed plane, the closed string twisted fields belong to linear multiplets, and its R-R part couples as Green-Schwarz terms to U(1) gauge fields and $F \wedge F$. By supersymmetry, we also expect FI couplings for its NS-NS partner. The $\mathcal{N}=2$ sectors are more involved. Actually, the ambiguity raised in [30], where they were unable to fix completely the anomalous couplings through the factorization approach, should be determined by the disk calculation. As a final comment, we evoke an open issue in these orientifold compactifications: the problem of target space duality symmetry [32, 29, 33] is not completely settled and needs more investigation.

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## A. Amplitude toolbox

**One-loop partition functions.** The contribution of the zero modes and oscillators of the spacetime coordinates and ghosts to the partition function, common to all the compactifications considered in this paper, is

$$Z_4^{\alpha,\beta}(\tau) = \frac{1}{4\pi^4\tau^2}\frac{\vartheta\left[\begin{smallmatrix}\alpha\\ \beta\end{smallmatrix}\right](0|\tau)}{\eta(\tau)^3} \tag{A.1}$$

for the annulus.
For the $K3\times\mathbb{T}^2$ orientifolds, the internal annulus partition function is:

$$Z_{\mathrm{int},k}^{\alpha,\beta}(\tau) = -\Gamma^{(2)}(\tau)\frac{\vartheta\left[\begin{smallmatrix}\alpha\\ \beta\end{smallmatrix}\right](0|\tau)}{\eta(\tau)^3}\left(2\sin\frac{\pi k}{N}\right)^2\prod_{j=1,2}\frac{\vartheta\left[\begin{smallmatrix}\alpha\\ \beta+kv_j\end{smallmatrix}\right](0|\tau)}{\vartheta\left[\begin{smallmatrix}1/2\\ 1/2+kv_j\end{smallmatrix}\right](0|\tau)}\,, \tag{A.2}$$

where $\Gamma^{(2)}(\tau)$ is the lattice sum over momenta along the untwisted two-torus:

$$\Gamma^{(2)}(\tau) = \sum_{n_4,n_5} e^{2i\pi\tau|n_4+n_5U|^2/(\sqrt{G}\,\mathrm{Im}\,U)} \tag{A.3}$$

with $G_{ab}$ $(a,b=4,5)$ the torus metric, and $U=(G_{45}+i\sqrt{G})/G_{44}$ its complex structure.
For the $\mathbb{T}^6/\mathbb{Z}_3$ model, the internal annulus partition function is:

$$Z_{\mathrm{int},k}^{\alpha,\beta}(\tau) = \prod_{j=1}^{3}(-2\sin\pi k v_j)\frac{\vartheta\left[\begin{smallmatrix}\alpha\\ \beta+kv_j\end{smallmatrix}\right](0|\tau)}{\vartheta\left[\begin{smallmatrix}1/2\\ 1/2+kv_j\end{smallmatrix}\right](0|\tau)}\,. \tag{A.4}$$

For the $\mathbb{T}^6/\mathbb{Z}_6'$ model, the internal annulus partition functions are:

$$\begin{aligned}
Z_{\mathrm{int},k}^{\alpha,\beta}(\tau) &= \prod_{j=1}^{3}(-2\sin\pi k v_j)\frac{\vartheta\left[\begin{smallmatrix}\alpha\\ \beta+kv_j\end{smallmatrix}\right](0|\tau)}{\vartheta\left[\begin{smallmatrix}1/2\\ 1/2+kv_j\end{smallmatrix}\right](0|\tau)}\,, \qquad k=1,5\,,\\
Z_{\mathrm{int},k}^{\alpha,\beta}(\tau) &= \Gamma_2^{(2)}(\tau)\frac{\vartheta\left[\begin{smallmatrix}\alpha\\ \beta+kv_2\end{smallmatrix}\right](0|\tau)}{\eta(\tau)^3}\prod_{j=1,3}(2\sin\pi k v_j)\frac{\vartheta\left[\begin{smallmatrix}\alpha\\ \beta+kv_j\end{smallmatrix}\right](0|\tau)}{\vartheta\left[\begin{smallmatrix}1/2\\ 1/2+kv_j\end{smallmatrix}\right](0|\tau)}\,, \qquad k=2,4\,,\\
Z_{\mathrm{int},k}^{\alpha,\beta}(\tau) &= \Gamma_3^{(2)}(\tau)\frac{\vartheta\left[\begin{smallmatrix}\alpha\\ \beta+3v_3\end{smallmatrix}\right](0|\tau)}{\eta(\tau)^3}\prod_{j=1,2}(2\sin 3\pi v_j)\frac{\vartheta\left[\begin{smallmatrix}\alpha\\ \beta+3v_j\end{smallmatrix}\right](0|\tau)}{\vartheta\left[\begin{smallmatrix}1/2\\ 1/2+3v_j\end{smallmatrix}\right](0|\tau)}\,, \qquad k=3\,.
\end{aligned} \tag{A.5}$$

The internal partition functions for the Möbius strip and the annulus with one boundary on a D9-brane and the other on a D5-brane can be found in appendix 2 of [7].

**One-loop correlation functions.** The bosonic correlation function on the torus $\mathcal{T}$ in the untwisted directions is:

$$\big\langle X(z_1)X(z_2)\big\rangle_{\mathcal{T}} = -\frac{1}{4}\ln\left|\frac{\vartheta_1(\nu_1-\nu_2|\tau)}{\vartheta_1'(0|\tau)}\right|^2 + \frac{\pi(\mathrm{Im}(\nu_1-\nu_2))^2}{2\,\mathrm{Im}\,\tau}\,, \tag{A.6}$$

The correlators on the annulus $\mathcal{A}$ and the Möbius strip $\mathcal{M}$ are obtained by symmetrizing this function under the involutions

$$I_{\mathcal{A}}(\nu) = I_{\mathcal{M}}(\nu) = 1 - \bar{\nu}\,. \tag{A.7}$$

The fermionic correlation functions on the torus are

$$\begin{aligned}
\left\langle \psi^\mu(z_1)\psi^\nu(z_2)\right\rangle_{\mathcal{T}}^{\alpha,\beta} &= \frac{i}{2}\frac{\vartheta\left[\begin{smallmatrix}\alpha\\ \beta\end{smallmatrix}\right](\nu_1-\nu_2|\tau)\vartheta_1'(0|\tau)}{\vartheta\left[\begin{smallmatrix}\alpha\\ \beta\end{smallmatrix}\right](0|\tau)\vartheta_1(\nu_1-\nu_2|\tau)}\delta^{\mu\nu}\,,\\
\left\langle \psi^i(z_1)\psi^j(z_2)\right\rangle_{\mathcal{T}}^{\alpha,\beta} &= \frac{i}{2}\frac{\vartheta\left[\begin{smallmatrix}\alpha\\ \beta+kv_i\end{smallmatrix}\right](\nu_1-\nu_2|\tau)\vartheta_1'(0|\tau)}{\vartheta\left[\begin{smallmatrix}\alpha\\ \beta+kv_i\end{smallmatrix}\right](0|\tau)\vartheta_1(\nu_1-\nu_2|\tau)}\delta^{i\bar{\jmath}}
\end{aligned} \tag{A.8}$$

for untwisted and twisted worldsheet fermions in the even spin structures. Like the boson propagators, the fermion propagators on the other surfaces can be determined from these correlators by the method of images (see appendix of [11] for more details).

The correlation function of two twisted fermions for strings with DN boundary conditions is

$$\left\langle \psi^i(z_1)\psi^j(z_2)\right\rangle_{\mathcal{A}_{95}}^{\alpha,\beta} = \frac{i}{2}\frac{\vartheta\left[\begin{smallmatrix}\alpha+1/2\\ \beta+kv_i\end{smallmatrix}\right](\nu_1-\nu_2|\tau)\vartheta_1'(0|\tau)}{\vartheta\left[\begin{smallmatrix}\alpha+1/2\\ \beta+kv_i\end{smallmatrix}\right](0|\tau)\vartheta_1(\nu_1-\nu_2|\tau)}\delta^{i\bar{\jmath}}\,. \tag{A.9}$$

**Theta function identity.**

$$\begin{aligned}
&\sum_{\substack{\alpha,\beta=0,1/2\\ \text{even}}}\frac{1}{2}\eta_{\alpha,\beta}\vartheta\begin{bmatrix}\alpha\\ \beta\end{bmatrix}(\nu|\tau)\vartheta\begin{bmatrix}\alpha+\delta_1\\ \beta+\gamma_1\end{bmatrix}(\nu|\tau)\vartheta\begin{bmatrix}\alpha+\delta_2\\ \beta+\gamma_2\end{bmatrix}(0|\tau)\vartheta\begin{bmatrix}\alpha+\delta_3\\ \beta+\gamma_3\end{bmatrix}(0|\tau)\\
&\quad=\frac{1}{2}\vartheta\begin{bmatrix}1/2\\ 1/2\end{bmatrix}(\nu|\tau)\vartheta\begin{bmatrix}1/2+\delta_1\\ 1/2+\gamma_1\end{bmatrix}(\nu|\tau)\vartheta\begin{bmatrix}1/2+\delta_2\\ 1/2+\gamma_2\end{bmatrix}(0|\tau)\vartheta\begin{bmatrix}1/2+\delta_3\\ 1/2+\gamma_3\end{bmatrix}(0|\tau)
\end{aligned} \tag{A.10}$$

valid for $\delta_1+\delta_2+\delta_3=0$ and $\gamma_1+\gamma_2+\gamma_3=0$.

**Tree-level twisted correlation functions.** The twist field correlators on the disk are:

$$\begin{aligned}
\left\langle \sigma_k\psi^m(z_1)\psi^i(x_1)\psi^j(x_2)\psi^n\sigma_k^\dagger(z_2)\right\rangle &= \left(\frac{z_1-x_1}{z_2-x_1}\right)^{kv_i}\left(\frac{z_1-x_2}{z_2-x_2}\right)^{kv_j}\times\\
&\times\left(\frac{\delta^{m\bar{\imath}}\delta^{j\bar{n}}}{(z_1-x_1)(x_2-z_2)}-\frac{\delta^{m\bar{\jmath}}\delta^{i\bar{n}}}{(z_1-x_2)(x_1-z_2)}+\frac{\delta^{m\bar{n}}\delta^{i\bar{\jmath}}}{(z_1-z_2)(x_1-x_2)}\right)
\end{aligned} \tag{A.11}$$

for the worldsheet fermions and

$$\begin{aligned}
\left\langle \sigma_k(z_1)\partial\phi^i(x_1)\partial\phi^j(x_2)\sigma_k^\dagger(z_2)\right\rangle &= \left(\frac{z_1-x_1}{z_2-x_1}\right)^{-(1-kv_i)}\left(\frac{z_1-x_2}{z_2-x_2}\right)^{-kv_i}\times\\
&\times\frac{\delta^{i\bar{\jmath}}}{(x_1-x_2)^2}\left[(1-kv_i)\frac{(z_1-x_1)}{(z_2-x_1)}+kv_i\frac{(z_1-x_2)}{(z_2-x_2)}\right]
\end{aligned} \tag{A.12}$$

for the bosons (up to a $(z_1 - z_2)$ dependent term which comes from the contraction of the twist fields on the disk, and is the same for (A.11) and (A.12). We also use the method of images to obtain the correlation functions of both left- and right-moving fields.

## References


[1] C. Angelantonj, M. Bianchi, G. Pradisi, A. Sagnotti and Y.S. Stanev, *Chiral asymmetry in four-dimensional open-string vacua*, *Phys. Lett.* **B 385** (1996) 96 [hep-th/9606169].

[2] Z. Kakushadze and G. Shiu, *4d chiral $N = 1$ type-I vacua with and without D5-branes*, *Nucl. Phys.* **B 520** (1998) 75 [hep-th/9706051];
G. Aldazabal, A. Font, L.E. Ibáñez and G. Violero, *$D = 4$, $N = 1$, type-IIB orientifolds*, *Nucl. Phys.* **B 536** (1998) 29 [hep-th/9804026];
G. Aldazabal, D. Badagnani, L.E. Ibáñez and A.M. Uranga, *Tadpole versus anomaly cancellation in $D = 4$, 6 compact IIB orientifolds*, *J. High Energy Phys.* **06** (1999) 031 [hep-th/9904071].

[3] V.S. Kaplunovsky, *One loop threshold effects in string unification*, hep-th/9205070.

[4] L.J. Dixon, V. Kaplunovsky and J. Louis, *Moduli dependence of string loop corrections to gauge coupling constants*, *Nucl. Phys.* **B 355** (1991) 649;
J.P. Derendinger, S. Ferrara, C. Kounnas and F. Zwirner, *On loop corrections to string effective field theories: field dependent gauge couplings and sigma model anomalies*, *Nucl. Phys.* **B 372** (1992) 145;
I. Antoniadis, K.S. Narain and T.R. Taylor, *Higher genus string corrections to gauge couplings*, *Phys. Lett.* **B 267** (1991) 37;
I. Antoniadis, E. Gava and K.S. Narain, *Moduli corrections to gauge and gravitational couplings in four-dimensional superstrings*, *Nucl. Phys.* **B 383** (1992) 93 [hep-th/9204030];
P. Mayr and S. Stieberger, *Threshold corrections to gauge couplings in orbifold compactifications*, *Nucl. Phys.* **B 407** (1993) 725 [hep-th/9303017];
H.P. Nilles and S. Stieberger, *String unification, universal one-loop corrections and strongly coupled heterotic string theory*, *Nucl. Phys.* **B 499** (1997) 3 [hep-th/9702110];
S. Stieberger, $(0, 2)$ *heterotic gauge couplings and their M-theory origin*, *Nucl. Phys.* **B 541** (1999) 109 [hep-th/9807124].

[5] I. Antoniadis and A. Sagnotti, *Mass scales, supersymmetry breaking and open strings*, *Class. and Quant. Grav.* **17** (2000) 939 [hep-th/9911205];
C.P. Bachas, *Scales of string theory*, *Class. and Quant. Grav.* **17** (2000) 1 [hep-th/0001093].

[6] C. Bachas and C. Fabre, *Threshold effects in open-string theory*, *Nucl. Phys.* **B 476** (1996) 418 [hep-th/9605028].

[7] I. Antoniadis, C. Bachas and E. Dudas, *Gauge couplings in four-dimensional type-I string orbifolds*, *Nucl. Phys.* **B 560** (1999) 93 [hep-th/9906039].

[8] E. Kiritsis, *Duality and instantons in string theory*, hep-th/9906018.

[9] E. Poppitz, *On the one loop Fayet-Iliopoulos term in chiral four dimensional type-I orbifolds*, *Nucl. Phys.* **B 542** (1999) 31 [hep-th/9810010].

[10] B. de Wit, P.G. Lauwers and A.V. Proeyen, *Lagrangians of $N = 2$ supergravity-matter systems*, *Nucl. Phys.* **B 255** (1985) 569.

[11] I. Antoniadis, C. Bachas, C. Fabre, H. Partouche and T.R. Taylor, *Aspects of type-I-type-II-heterotic triality in four dimensions*, *Nucl. Phys.* **B 489** (1997) 160 [hep-th/9608012].

[12] M.R. Douglas and G. Moore, *D-branes, quivers and ALE instantons*, hep-th/9603167.

[13] A. Abouelsaood, C.G. Callan, C.R. Nappi and S.A. Yost, *Open strings in background gauge fields*, *Nucl. Phys.* **B 280** (1987) 599;
C. Bachas and M. Porrati, *Pair creation of open strings in an electric field*, *Phys. Lett.* **B 296** (1992) 77 [hep-th/9209032].

[14] E.G. Gimon and J. Polchinski, *Consistency conditions for orientifolds and D-manifolds*, *Phys. Rev.* **D 54** (1996) 1667 [hep-th/9601038].

[15] E.G. Gimon and C.V. Johnson, *K3 orientifolds*, *Nucl. Phys.* **B 477** (1996) 715 [hep-th/9604129].

[16] T. Takayanagi, *String creation and monodromy from fractional D-branes on ALE spaces*, *J. High Energy Phys.* **02** (2000) 040 [hep-th/9912157].

[17] E. Kiritsis and C. Kounnas, *Infrared regularization of superstring theory and the one loop calculation of coupling constants*, *Nucl. Phys.* **B 442** (1995) 472 [hep-th/9501020].

[18] I. Antoniadis, E. Gava, K.S. Narain and T.R. Taylor, *Superstring threshold corrections to Yukawa couplings*, *Nucl. Phys.* **B 407** (1993) 706 [hep-th/9212045].

[19] J.J. Atick, L.J. Dixon and A. Sen, *String calculation of Fayet-Iliopoulos D terms in arbitrary supersymmetric compactifications*, *Nucl. Phys.* **B 292** (1987) 109;
M. Dine, I. Ichinose and N. Seiberg, *F-terms and D-terms in string theory*, *Nucl. Phys.* **B 293** (1987) 253.

[20] V. Kaplunovsky and J. Louis, *On gauge couplings in string theory*, *Nucl. Phys.* **B 444** (1995) 191 [hep-th/9502077].

[21] L. Dixon, D. Friedan, E. Martinec and S. Shenker, *The conformal field theory of orbifolds*, *Nucl. Phys.* **B 282** (1987) 13.

JHEP04(2000)013

[22] G. Pradisi and A. Sagnotti, *Open string orbifolds*, *Phys. Lett.* **B 216** (1989) 59;
M. Bianchi and A. Sagnotti, *On the systematics of open string theories*, *Phys. Lett.* **B 247** (1990) 517; *Twist symmetry and open string Wilson lines*, *Nucl. Phys.* **B 361** (1991) 519.

[23] A. Dabholkar and J. Park, *An orientifold of type-IIB theory on $k3$*, *Nucl. Phys.* **B 472** (1996) 207 [hep-th/9602030];
J.D. Blum, *F-theory orientifolds, M-theory orientifolds and twisted strings*, *Nucl. Phys.* **B 486** (1997) 34 [hep-th/9608053].

[24] J. Polchinski, *Tensors from K3 orientifolds*, *Phys. Rev.* **D 55** (1997) 6423 [hep-th/9606165].

[25] D.-E. Diaconescu and J. Gomis, *Fractional branes and boundary states in orbifold theories*, hep-th/9906242.

[26] N. Marcus and A. Sagnotti, *Group theory from 'quarks' at the ends of strings*, *Phys. Lett.* **B 188** (1987) 58.

[27] P. West, *The Yukawa beta function in $N = 1$ rigid supersymmetric theories*, *Phys. Lett.* **B 137** (1984) 371;
A. Parkes and P. West, *Finiteness in rigid supersymmetric theories*, *Phys. Lett.* **B 138** (1984) 99.

[28] C.A. Scrucca and M. Serone, *Anomaly cancellation in K3 orientifolds*, *Nucl. Phys.* **B 564** (2000) 555 [hep-th/9907112].

[29] M. Klein, *Anomaly cancellation in $D = 4$, $N = 1$ orientifolds and linear/chiral multiplet duality*, *Nucl. Phys.* **B 569** (2000) 362 [hep-th/9910143].

[30] C.A. Scrucca and M. Serone, *Gauge and gravitational anomalies in $d = 4$ $N = 1$ orientifolds*, *J. High Energy Phys.* **12** (1999) 024 [hep-th/9912108].

[31] C.P. Bachas, *Unification with low string scale*, *J. High Energy Phys.* **11** (1998) 023 [hep-ph/9807415].

[32] L.E. Ibáñez, R. Rabadan and A.M. Uranga, *Sigma-model anomalies in compact $D = 4$, $N = 1$ type-IIB orientifolds and Fayet-Iliopoulos terms*, hep-th/9905098.

[33] Z. Lalak, S. Lavignac and H.P. Nilles, *Target-space duality in heterotic and type-I effective lagrangians*, hep-th/9912206.